12.1

a) we begin with the time-independent Schrödinger equation with a delta potential:

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V\right]\psi = E\psi$$

This can be rewritten as:

$$\left[E + \frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right]\psi(x) = \sum_{n=-\infty}^{\infty} V_0\,\delta(x - n\Delta)\,\psi(x)$$

Let's integrate from $x = -\epsilon$ to $x = \epsilon$ and take the limit $\epsilon \to 0$:

$$\int_{-\epsilon}^{\epsilon} E\psi(x)\,dx = \int_{-\epsilon}^{\epsilon} -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}\,dx + \int_{-\epsilon}^{\epsilon} \sum_{n=-\infty}^{\infty} V_0\,\delta(x - n\Delta)\,\psi(x)\,dx$$

As $\epsilon \to 0$, the first term on left side approaches zero because the integral of $\psi(x)$ over an infinitesimal region is infinitesimal:

$$\lim_{\epsilon \to 0} \int_{-\epsilon}^{\epsilon} E\psi(x)\,dx = 0$$

for the first term on the right side, we can use integration by parts:

$$\int_{-\epsilon}^{\epsilon} \frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2}\,dx = \frac{-\hbar^2}{2m}\left[\frac{d\psi}{dx}\right]_{-\epsilon}^{\epsilon} - \int_{-\epsilon}^{\epsilon} \frac{-\hbar^2}{2m}\frac{d}{dx}\frac{d\psi}{dx}\,dx$$

The second term in this expression vanishes as $\epsilon \to 0$, leaving:

$$\lim_{\epsilon \to 0} \int_{-\epsilon}^{\epsilon} \frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2}\,dx = \frac{-\hbar^2}{2m}\left[\frac{d\psi}{dx}\bigg|_{\epsilon} - \frac{d\psi}{dx}\bigg|_{-\epsilon}\right]$$

For the delta function term, since we're integrating around $x=0$, only the $n=0$ term contributes:

$$\lim_{\epsilon \to 0} \int_{-\epsilon}^{\epsilon} \sum_{n=-\infty}^{\infty} V_0 \delta(x - n\Delta)\psi(x)dx = V_0 \psi(0)$$

Combining these results,

$$\frac{\hbar^2}{2m}\left[\left.\frac{d\psi}{dx}\right|_{\epsilon} - \left.\frac{d\psi}{dx}\right|_{-\epsilon}\right] = V_0 \psi(0)$$

The equation represents the boundary condition at $x=0$ due to the delta potential. It shows that while the wave function $\psi(x)$ is continuous at $x=0$, its derivative has a discontinuity proportional to the strength of the potential V_0.

Now, we know that the wavefunc. has the form:

$$\psi(x) = Ae^{iqx} + Be^{-iqx}$$

This represents a superposition of waves travelling in opposite directions, which is the general solution to the Schrödinger equation in regions of constant potential.

To apply the boundary condition, I need to calculate the derivatives at $x=\epsilon$ and $x=-\epsilon$. Taking the derivative:

$$\frac{d}{dx}(Ae^{iqx} + Be^{-iqx}) = iqAe^{iqx} - iqBe^{-iqx}$$

At $x=\epsilon$ (approaching from the right):

$$\left.\frac{d\psi}{dx}\right|_{\epsilon} = iqAe^{iq(qn-k\Delta)} - iqBe^{-iq(qn+k\Delta)}$$

At $x = -\varepsilon$ (approaching from the left):

$$\left.\frac{d\psi}{dx}\right|_{-\varepsilon} = iqAe^{iqn} - iqBe^{-iqn}$$

Substituting these into our boundary condition,

$$\frac{\hbar^2}{2m}\left[iqAe^{iq(qn-k\Delta)} - iqBe^{-iq(qn+k\Delta)} - (iqAe^{iqn} - iqBe^{-iqn})\right] = V_0\psi(0)$$

Simplifying:

$$\frac{\hbar^2}{2m} iq\left[A\left(e^{iq(qn-k\Delta)} - e^{iqn}\right) - B\left(e^{-iq(qn+k\Delta)} - e^{-iqn}\right)\right] = V_0(A+B)$$

This gives us equation:

$$\frac{\hbar^2}{2m} iq\left(A - B - Ae^{i(qn-k\Delta)} + Be^{-i(qn+k\Delta)}\right) = V_0(A+B)$$

42.2

The Fermi-Dirac distribution function describes the probability of occupancy of energy states by fermions at thermal equilibrium:

$$f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

$$E_f = \frac{E_c + E_v}{2} = E_c - \frac{E_g}{2}$$

where E_g is the bandgap energy.

Thermal Energy kT at room Temperature:

$$kT = 8.617 \times 10^{-5}\ eV/K \times 300K = 0.026\ eV$$

Germanium (Ge): The bandgap of Ge is 0.67 eV, so

$$E_f = E_c - \frac{0.67\ eV}{2} = E_c - 0.335\ eV$$

Substituting into the Fermi-Dirac distribution.

$$f(E_c) = \frac{1}{1+e^{(E_c-(E_c-0.335eV))/0.026eV}} = \frac{1}{1+e^{0.335/0.026}}$$

Calculating:

$$f(E_c) = \frac{1}{1+e^{12.88}} = \frac{1}{1+3.92\times10^5} = 2.55\times10^{-6}$$

Silicon(Si): The bandgap of Si is 1.11 eV, so:

$$E_f = E_c - \frac{1.11\ eV}{2} = E_c - 0.555\ eV$$

Substituting into the Fermi-Dirac distribution:

$$f(E_c) = \frac{1}{1+e^{(E_c-(E_c-0.555eV))/0.026eV}} = \frac{1}{1+e^{0.555/0.026}}$$

Calculating:

$$f(E_c) = \frac{1}{1+e^{21.35}} = \frac{1}{1+1.88\times10^{9}} = 5.32\times10^{-10}$$

Diamond. The bandgap of diamond is 5.5 eV, so:

$$E_F = E_c - \frac{5.5\,eV}{2} = E_c - 2.75\,eV$$

Substituting into the fermi-Dirac distribution:

$$f(E_c) = \frac{1}{1+e^{(E_c-(E_c-2.75eV))/0.026\,eV}} = \frac{1}{1+e^{2.75/0.026}}$$

Calculating:

$$f(E_c) = \frac{1}{1+e^{105.77}} = \frac{1}{1+1.39\times10^{46}} = 7.20\times10^{-47}$$

12.3

a)

$$N_C = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$$

Where:- m_e^* is the effective mass of electrons in silicon - k is Boltzmann's constant - T is temperature (300k) - h is Planck's constant.

Substituting the values:

$$N_C = 2\left(\frac{2\pi \times 1.1 \times 9.1095 \times 10^{-31}\,kg \times 1.3807 \times 10^{-23}\,J/K \times 300\,K}{(6.626 \times 10^{-34}\,J.s)^2}\right)^{3/2}$$

Calculating:

$$N_C = 2\left(\frac{2\pi \times 1.1 \times 9.1095 \times 10^{-31} \times 1.3807 \times 10^{-23} \times 300}{(6.626 \times 10^{-34})^2}\right)^{3/2} = 2.895 \times 10^{19}\,cm^{-3}$$

Similarly, for the effective density of states in the valence band:

$$N_V = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

Where m_h^* is the effective mass of holes in silicon, which is approximately 0.56 times the electron mass.

Substituting

$$N_V = 2\left(\frac{2\pi \times 0.56 \times 9.1095 \times 10^{-31}\,kg \times 1.3807 \times 10^{-23}\,J/K \times 300K}{(6.626 \times 10^{-34}\,J.S)^2}\right)^{3/2}$$

Calculating:

$$N_V = 2\left(\frac{2\pi \times 0.56 \times 9.1095 \times 10^{-31} \times 1.3807 \times 10^{-23} \times 300}{(6.626 \times 10^{-34})^2}\right)^{3/2} = 1.052 \times 10^{19}\,cm^{-3}$$

Now, the intrinsic carrier concentration n_i can be calculated using

$$n_i^2 = N_C N_V e^{-E_g/kT}$$

Where E_g is the bandgap of silicon (1.11eV).

Converting the bandgap to joules:

$$E_g = 1.11eV \times 1.602 \times 10^{-19} J/eV = 1.778 \times 10^{-19} J$$

Substituting:

$$n_i^2 = 2.895 \times 10^{19} cm^{-3} \times 1.052 \times 10^{19} cm^{-3} \times e^{-1.778 \times 10^{-19} / (1.3807 \times 10^{-23} \times 300)}$$

Calculating:

$$n_i^2 = 2.895 \times 10^{19} \times 1.052 \times 10^{19} \times e^{-42.97} = 3.045 \times 10^{38} \times 1.97 \times 10^{-19} = 6.00 \times 10^{19} cm^{-6}$$

$$n_i = \sqrt{6.00 \times 10^{19}} = 7.75 \times 10^{9} cm^{-3}$$

In an n-type semiconductor (doped with donors like As), the electron concentration n is approximately equal to the donor concentration N_d when $N_d \gg n_i$:

$$n \approx N_d = 10^{17} cm^{-3}$$

Using the mass action law, which state that $np = n_i^2$ in thermal equilibrium:

~~$p = \frac{n_i^2}{0}$ in thermal equilibrium:~~

$$p = \frac{n_i^2}{n} = \frac{6.00 \times 10^{19} cm^{-6}}{10^{17} cm^{-3}} = 6.00 \times 10^{2} cm^{-3}$$

(b) The Fermi level in a doped semiconductor shifts from its intrinsic position. For an n-type semiconductor, the Fermi level moves closer to the conduction band. The relationship between the electron concentration and the Fermi level is given by:

$$n = n_i e^{(E_f - E_i)/kT}$$

Where $-E_f$ is the Fermi level in the doped semiconductor
$-E_i$ is the intrinsic Fermi level $-kT$ is the thermal energy
Rearranging to find the shift in Fermi level:

$$E_F - E_i = kT \ln\left(\frac{n}{n_i}\right)$$

Substituting the values:

$$E_F - E_i = 8.617 \times 10^{-5}\ eV/K \times 300\ K \times \ln\left(\frac{10^{17}\ cm^{-3}}{7.75 \times 10^{9}\ cm^{-3}}\right)$$

Calculating:

$$E_F - E_i = 0.026\ eV \times \ln\left(\frac{10^{17}}{7.75 \times 10^{9}}\right) = 0.026\ eV \times \ln\left(1.29 \times 10^{7}\right)$$

$$= 0.026\ eV \times 16.37 = 0.426\ eV$$

12.5

a) The energy stored in a capacitor is given by:

$$E_{stored} = \frac{1}{2} CV^2$$

Where: -C is capacitance -V is the voltage across the capacitor

Given that $C = 1 \times 10^{-15}$ F and $V = 1.8$V, the stored energy is

$$E_{stored} = \frac{1}{2} \times 1 \times 10^{-15} F \times (1.8V)^2 = \frac{1}{2} \times 1 \times 10^{-15} \times 3.24$$

$$= 1.62 \times 10^{-15} J$$

b) The current in an RC circuit during discharge is given by:

$$I = \frac{V - V_C}{R} = C \frac{dV_C}{dt} = \frac{1}{RC}(V - V_C)$$

Where: -V is the initial voltage -V_C is the capacitor voltage at time t -R is the resistance. This differential equation has the solution:

$$V_C = V(1 - e^{-t/RC})$$

For a discharging capacitor (initially charged to voltage V), the solution is:

$$V_C = Ve^{-t/RC}$$

The current is then

$$I = \frac{V}{R} e^{-t/RC}$$

The power dissipated in the resistor at any time is:

$$P = I^2 R = \frac{V^2}{R} e^{-2t/RC}$$

The total energy dissipated is the integral of power over time:

$$E_{dissipated} = \int_0^\infty P\,dt = \int_0^\infty \frac{V^2}{R} e^{-2t/RC}\,dt$$

Solving this integral:

$$E_{dissipated} = \frac{V^2}{R}\int_0^\infty e^{-2t/RC}\,dt = \frac{V^2}{R}\left[-\frac{RC}{2}e^{-2t/RC}\right]_0^\infty$$

$$= \frac{V^2}{R}\cdot\frac{RC}{2} = \frac{1}{2}CV^2$$

c) During the ramping period, the differential equation for the capacitor voltage becomes:

$$\frac{dV_C}{dt} = \frac{1}{RC}\left(\frac{V}{T}t - V_C\right)$$

where $\frac{V}{T}t$ represents the linearly increasing supply voltage

To solve this, I'll try an ansatz (educated guess) of the form:

$$V_C = \frac{V}{T}t + A$$

where A is a constant to be determined.

Substituting this into the differential equation:

$$\frac{V}{T} = \frac{1}{RC}\left(\frac{V}{T}t - \left(\frac{V}{T}t + A\right)\right) = \frac{1}{RC}(-A)$$

solving for A:

$$\frac{V}{T} = -\frac{A}{RC}$$

Therefore: $A = -\frac{RCV}{T}$

This means the capacitor voltage lags behind the supply voltage by a constant amount proportional to RC/T:

$$V_C = \frac{V}{T}t - \frac{RCV}{T}$$

The voltage difference across the resistor is:

$$V_R = \frac{V}{T}t - V_C = \frac{RCV}{T}$$

This is constant throughout the ramping period, which means the current is also constant:

$$I = \frac{V^2}{R} = \frac{CV}{T}$$

The power dissipated in the resistor is:

$$P = I^2R = \left(\frac{CV}{T}\right)^2 R = \frac{C^2V^2R}{T^2}$$

The total energy dissipated during the ramping period is:

$$E_{dissipated} = \int_0^T P\,dt = \int_0^T \frac{C^2V^2R}{T^2}\,dt = \frac{C^2V^2R}{T^2} \cdot T = \frac{C^2V^2R}{T}$$

12.5 (d)

$$10^{-15}\,F \times (1.8V)^2 = 3.24 \times 10^{-15}\,J$$

$$\frac{1\,J \cdot s^{-1}}{3.24 \times 10^{-15}\,J} = 3 \times 10^{14}\,Hz$$

12.5 (e)

$$3.24 \times 10^{-15}\,J \times 10^{9}\ \text{transistors} \times 10^{9}\,Hz = 3240\,W$$

12.5 (f) $Q = CV = 10^{-15}\,F \times 1.8V = 1.8 \times 10^{-15}\,C$

$$1.8 \times 10^{-15}\,C \times \frac{1\ \text{electron}}{1.6 \times 10^{-19}\,C} = 1.1 \times 10^{4}\ \text{electrons}$$